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Theory I - Exercise sheet 5

Submission due **Tuesday, July 7**

Exercise 1: (4 points) Membership-Test
The following axioms are considered ($r \in SAT(V, \mathcal{F})$):

Reflexivity (A1) $Y \subseteq X \subseteq V \Longrightarrow r$ satisfies also $X \to Y$.

Augmentation(A2) r satisfies $X \to Y$, $Z \subseteq V \Longrightarrow r$ satisfies also $XZ \to YZ$.

Transitivity (A3) r satisfies $X \to Y, Y \to Z \Longrightarrow r$ satisfies $X \to Z$.

Decomposition (A6) r satisfies $X \to Y$, $Z \subseteq Y \Longrightarrow r$ satisfies also $X \to Z$

Reflexivity (A7) r satisfies $X \to X$

Accumulation (A8) r satisfies $X \to YZ, Z \to AW \Longrightarrow r$ satisfies also $X \to YZA$

Show the equivalence of the axiomatic systems (A1) – (A3) and (A6) – (A8).

Exercise 2: (6 points) BCNF
$R = (V, \mathcal{F})$ is a relational schema. Show the following coherences:

a) R is in BCNF, iff for every non-trivial functional dependency $X \to A \in \mathcal{F}^+$ X is a superkey.

b) R is in BCNF, iff $R' = (V, \mathcal{F}^+)$ is in BCNF.

c) R has exactly one key. R is in BCNF, iff R is in 3NF.

d) $\mathcal{F} = \{X_1 \to Y_1, \ldots, X_p \to Y_p\}$. R has an unambiguous key, iff $V \setminus Z_1 \ldots Z_p$ is a superkey, whereas $Z_i = Y_i \setminus X_i$, $1 \leq i \leq p$.

Exercise 3: (4 points) Complexity of Functional Dependencies

a) Show that sets of attributes V, $X \subseteq V$ and a set of functional dependencies $\mathcal{F}$ exist, where $\mathcal{F}$ contains $2n + 1$ functional dependencies, such that $\pi[x]\mathcal{F}$ has at least 2^n elements.

b) Discuss the consequences of a).

Exercise 4: (6 points) Formal Design

$R(A, B, C)$ is a relational schema and r a relation to R.

a) Give an r such that $r \models X \rightarrow Y$ iff $X \rightarrow Y$ is trivial.

b) Given $\mathcal{F} = \{A \rightarrow B, B \rightarrow C\}$. Give an r such that $r \models X \rightarrow Y$ iff $X \rightarrow Y \in \mathcal{F}^+$.

c) Given $\mathcal{F} = \{\emptyset \rightarrow A, \emptyset \rightarrow C\}$. Give a relation r which does not fulfill $\mathcal{F}$.

d) Given $\mathcal{F} = \{A \rightarrow \emptyset, C \rightarrow \emptyset\}$. Give a relation r which fulfills $\mathcal{F}$.